De-coherence and transition from Quantum to Classical

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March 22, 2013

Outline

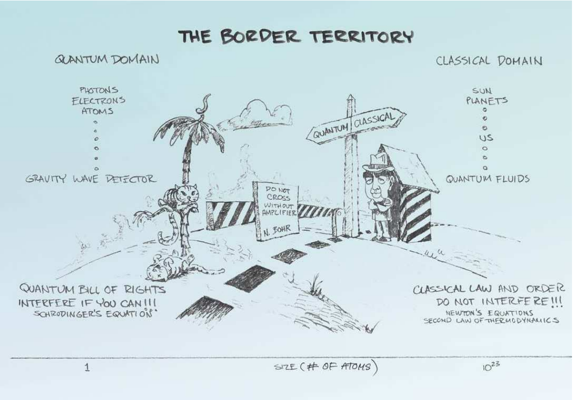
Introduction

- Quantum Mechanics has been the most successful theory.
- The only "failure" of the theory is its inability to provide a natural framework for our prejudices about working of this universe.
- Schröndinger's equation

$$\iota\hbar\frac{d|\Psi\rangle}{dt} = \hat{H}|\Psi\rangle. \tag{1}$$

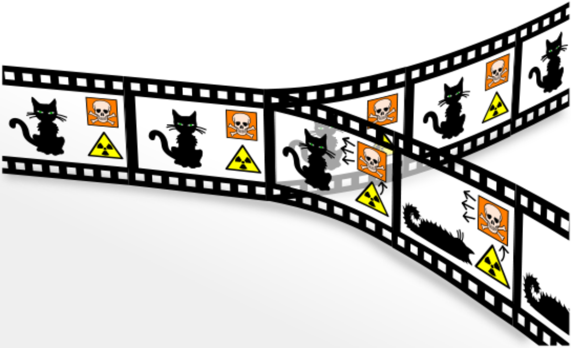
- Although it is conciousness that registers only one of the many choices, there is every indication that choice is made much before the conciousness gets involved and once made, the choice is irrevocable.

Interpretations of Quantum Mechanics



- Copenhagen (Niels Henrik David Bohr)
  - Apparatus is Classical.
  - System is Quantum Mechanical.
  - There is a movable dividing line between Quantum and Classical domain.

Interpretations of Quantum Mechanics



- Many Universes (Hugh Everett III with encouragement from John Archibald Wheeler)
    - Superpositions evolve forever according to the Schrödinger equation.
    - Each time a suitable interaction takes place between any two quantum systems, the wave function of the universe splits, developing ever more “branches”.

- Why don't we see the coherent superposition of macroscopic systems?
- De-coherence is the culprit which destroys "Quantumness".

  De-coherence imposes, in effect, the required "embargo" on the potential outcomes by allowing the observer to maintain records of alternatives but to be aware of only one of the branches.[Zurek, 1991]

Quantum Measurements

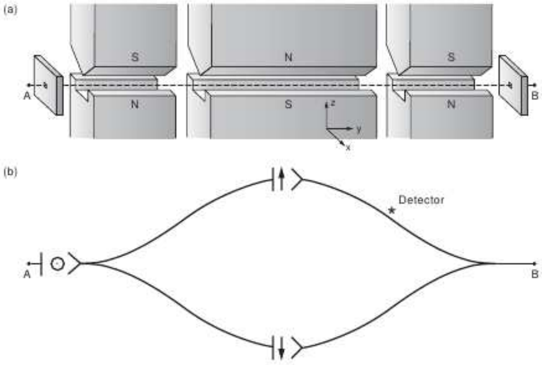


Figure : Stern-Gerlach apparatus

- Let the initial state vector be given by

$$|\psi^i\rangle = (\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes |d_\downarrow\rangle \tag{2}$$

- Under a suitable interaction, the state vector of the composite system can be given as

$$|\varphi^c\rangle = \alpha|\uparrow\rangle|d_\uparrow\rangle + \beta|\downarrow\rangle|d_\downarrow\rangle \tag{3}$$

- $|\varphi^c\rangle$ is not a suitable candidate for the description of completed measurement.
  - $|\varphi^c\rangle$ is an EPR state.
  - It gives the complete information of the composite system but not of individual systems.
  - The states of the two spins in a system described by $|\varphi^c\rangle$ are not just unknown, but rather *they cannot exist before the "real" measurement.*
  - There is no unique expansion. This is called *The problem of preferred basis.*

John von Neumann's postulate

- Consider a density matrix corresponding to pure state $|\varphi^c\rangle$ written as

$$\begin{aligned}\hat{\rho}^c &= |\varphi^c\rangle\langle\varphi^c| \qquad (4)\\ &= |\alpha|^2|\uparrow\rangle\langle\uparrow||d_\uparrow\rangle\langle d_\uparrow| + |\beta|^2|\downarrow\rangle\langle\downarrow||d_\downarrow\rangle\langle d_\downarrow| \\ &\quad + \alpha\beta^*|\uparrow\rangle\langle\downarrow||d_\uparrow\rangle\langle d_\downarrow| + \beta\alpha^*|\downarrow\rangle\langle\uparrow||d_\downarrow\rangle\langle d_\uparrow| \qquad (5)\end{aligned}$$

- von Neumann postulated an ad-hoc "process 1", a nonunitary reduction of the state vector, that would take the pure, correlated density matrix $\hat{\rho}^c$ into an appropriate mixture $\hat{\rho}^r$ obtained by cancelling off-diagonal terms. Hence

$$\hat{\rho}^r = |\alpha|^2|\uparrow\rangle\langle\uparrow||d_\uparrow\rangle\langle d_\uparrow| + |\beta|^2|\downarrow\rangle\langle\downarrow||d_\downarrow\rangle\langle d_\downarrow| \qquad (6)$$

- $\hat{\rho}^r$ is a suitable candidate for the description of completed measurement.
  - It shows that the system and the detector are in a definite and classically correlated but unknown state.
  - $\hat{\rho}^r$ reflects our classical ignorance.

Example

Consider a split coin put in two sealed envelopes (one envelope containing head and other containing tail). The system is classically correlated and the probabilities reflect our ignorance. It can safely be said that one envelope contains either heads or tails (and not some superimposition) and if one envelope contains heads, the other is bound to contain tails.

Missing Information and Decoherence

- In the previous sections, quantum correlation was analysed from the point of view of its role in acquiring information.
- Quantum correlations can also disperse information throughout the degrees of freedom that are, in effect, inaccessible to the observer.
- Interaction with the degrees of freedom external to the system, which we shall summarily refer to as the *environment*, offers such a possibility.
- Reduction of the state vector, $\hat{\rho}^c \Rightarrow \hat{\rho}^r$, decreases the information available to the observer, about the composite system $\mathcal{SD}$, which is necessary for the outcomes to become "classical".

- The entropy $\mathcal{H}$ of a system is defined as

$$\mathcal{H} = -tr(\hat{\rho} \ln \hat{\rho}) \tag{7}$$

- Then the increase in entropy of process $\hat{\rho}^c \Rightarrow \hat{\rho}^r$ is given as

$$\begin{aligned} \Delta\mathcal{H} &= \mathcal{H}(\hat{\rho}^r) - \mathcal{H}(\hat{\rho}^c) \\ &= -(|\alpha|^2 \ln |\alpha|^2 + |\beta|^2 \ln |\beta|^2) \end{aligned} \tag{8}$$

- The entropy must increase because the system was earlier in pure state and later turned into mixed state.

  Information gain, the objective of measurement, is accomplished only when observer interacts with the system and becomes co-related with the detector in already pre-collapsed state $\hat{\rho}^r$*.[Zurek, 1991]*

Environment induced de-coherence

- Consider a system $\mathcal{S}$, a detector $\mathcal{D}$ and an environment $\mathcal{E}$.*All of them are quantum mechanical.*
- After the establishment of co-relation, between system and detector, the environment $\mathcal{E}$ interacts with the composite system $\mathcal{SD}$.

$$|\varphi^c\rangle \otimes |\mathcal{E}_0\rangle = (\alpha|\uparrow\rangle|d_\uparrow\rangle + \beta|\downarrow\rangle|d_\downarrow\rangle)|\mathcal{E}_0\rangle \tag{9}$$

- Again, under suitable interaction

$$|\Psi\rangle = \alpha|\uparrow\rangle|d_\uparrow\rangle|\mathcal{E}_\uparrow\rangle + \beta|\downarrow\rangle|d_\downarrow\rangle|\mathcal{E}_\downarrow\rangle \tag{10}$$

- The combined co-relation of $\mathcal{SDE}$ (von Neumann chain) extends the co-relation beyond the composite system $\mathcal{SD}$ to the environment.
- When $\langle\mathcal{E}_\downarrow|\mathcal{E}_\uparrow\rangle = \delta_{\downarrow\uparrow}$, the density matrix for the detector-system combination is obtained by the partial trace (over the degrees of freedom of the environment).

$$
\begin{aligned}
\hat{\rho}^{\mathcal{SDE}} &= |\Psi\rangle\langle\Psi| \\
&= \Big(\alpha|\uparrow\rangle|d_\uparrow\rangle|\mathcal{E}_\uparrow\rangle + \beta|\downarrow\rangle|d_\downarrow\rangle|\mathcal{E}_\downarrow\rangle\Big)\Big(\alpha^*\langle\uparrow|\langle d_\uparrow|\langle\mathcal{E}_\uparrow| + \beta^*\langle\downarrow|\langle d_\downarrow|\langle\mathcal{E}_\downarrow|\Big) \\
&= |\alpha|^2|\uparrow\rangle|d_\uparrow\rangle|\mathcal{E}_\uparrow\rangle\langle\uparrow|\langle d_\uparrow|\langle\mathcal{E}_\uparrow| + |\beta|^2|\downarrow\rangle|d_\downarrow\rangle|\mathcal{E}_\downarrow\rangle\langle\downarrow|\langle d_\downarrow|\langle\mathcal{E}_\downarrow| \\
&\quad + \alpha\beta^*|\uparrow\rangle|d_\uparrow\rangle|\mathcal{E}_\uparrow\rangle\langle\downarrow|\langle d_\downarrow|\langle\mathcal{E}_\downarrow| + \alpha^*\beta|\downarrow\rangle|d_\downarrow\rangle|\mathcal{E}_\downarrow\rangle\langle\uparrow|\langle d_\uparrow|\langle\mathcal{E}_\uparrow| \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
\hat{\rho}^{\mathcal{SD}} &= tr_{\mathcal{E}}(\hat{\rho}^{\mathcal{SDE}}) \\
&= tr_{\mathcal{E}}\Big(|\alpha|^2|\uparrow\rangle|d_\uparrow\rangle|\mathcal{E}_\uparrow\rangle\langle\uparrow|\langle d_\uparrow|\langle\mathcal{E}_\uparrow| + |\beta|^2|\downarrow\rangle|d_\downarrow\rangle|\mathcal{E}_\downarrow\rangle\langle\downarrow|\langle d_\downarrow|\langle\mathcal{E}_\downarrow| \\
&\quad + \alpha\beta^*|\uparrow\rangle|d_\uparrow\rangle|\mathcal{E}_\uparrow\rangle\langle\downarrow|\langle d_\downarrow|\langle\mathcal{E}_\downarrow| + \alpha^*\beta|\downarrow\rangle|d_\downarrow\rangle|\mathcal{E}_\downarrow\rangle\langle\uparrow|\langle d_\uparrow|\langle\mathcal{E}_\uparrow|\Big) \\
&= |\alpha|^2|\uparrow\rangle\langle\uparrow||d_\uparrow\rangle\langle d_\uparrow| + |\beta|^2|\downarrow\rangle\langle\downarrow||d_\downarrow\rangle\langle d_\downarrow| \qquad (12)
\end{aligned}
$$

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