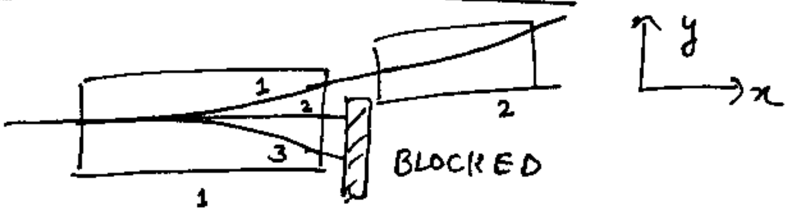
QUANTUM MECHANICS (MATRIX TREATMENT)
VISUALIZATION

SPIN ONE

(Sole purpose is to visualise Bra-Ket not to prove anything)

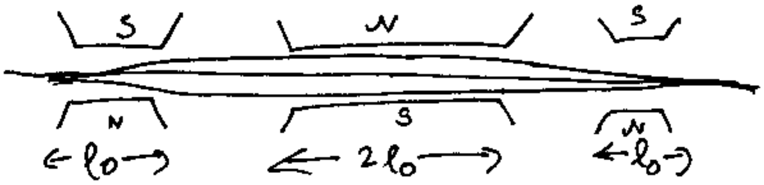
STERN-GERLACH APPARATUS



apparatus 1 and 2 are in same plane

We get 3 varieties of beams for spin one particles. By blocking ② and ③ beams we get beam ① and we can predict its future behaviour

MODIFICATION



S

$\cong \begin{Bmatrix} + \\ 0 \\ - \end{Bmatrix} S$ these are certain states w.r.t apparatus S

From earlier figure

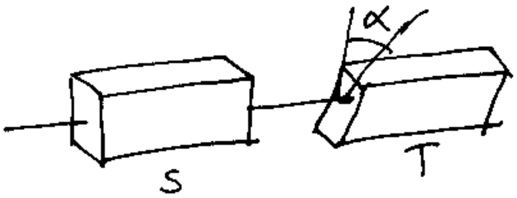
$$\boxed{\langle +S \mid +S \rangle = 1 \qquad \langle 0S \mid +S \rangle = 0 \qquad \langle -S \mid +S \rangle = 0}$$

$\langle +S \mid 0S \rangle = 0$ and etc

$$|||$$

$$\begin{array}{c|ccc} & + & 0 & - \\ \hline + & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ - & 0 & 0 & 1 \end{array}$$

EXPERIMENTS WITH TILTED | FILTERED APPARATUS



If atoms are in some state eg (+S) then they are not in (+T) state. Rather they are in some linear combination of states (w.r.t T). Explained later

∴ There is some amplitude that they go in +T, or 0T or −T states defined by

$$\langle +T \mid +S \rangle \quad \text{or} \quad \langle -T \mid +S \rangle \quad \text{etc}$$

★ The laws of Quantum Mechanics permit us to determine amplitude that a particle will get through certain apparatus.

We calculate all the possible amplitudes

$$\begin{array}{ccc} \langle +T \mid +S \rangle & \langle +T \mid 0S \rangle & \langle +T \mid -S \rangle \\ \langle 0T \mid +S \rangle & \langle 0T \mid 0S \rangle & \langle 0T \mid -S \rangle \\ \langle -T \mid +S \rangle & \langle -T \mid 0S \rangle & \langle -T \mid -S \rangle \end{array}$$

Let us ~~assume~~ consider Two Stern Gerlach Apparatus

(1) $\underset{S}{\begin{Bmatrix} + \\ 0 \\ - \end{Bmatrix}}$ $\underset{T}{\begin{Bmatrix} + \\ 0 \\ - \end{Bmatrix}}$ (They are tilted at α)

$$\langle +T \mid +S \rangle \langle +T \mid +S \rangle^* + \langle 0T \mid +S \rangle \langle 0T \mid +S \rangle^* + \langle -T \mid +S \rangle \langle -T \mid +S \rangle^* = 1$$

In contrast to this

(2) $\underset{S}{\begin{Bmatrix} + \\ 0 \\ - \end{Bmatrix}}$ $\underset{T}{\begin{Bmatrix} + \\ 0 \\ - \end{Bmatrix}}$ $\underset{S}{\begin{Bmatrix} + \\ 0 \\ - \end{Bmatrix}}$

$$\langle +S \mid +T \rangle \langle +T \mid +S \rangle + \langle +S \mid 0T \rangle \langle 0T \mid +S \rangle + \langle +S \mid -T \rangle \langle -T \mid +S \rangle = 1$$ (comes out 1 shown later)

In case I: I am at T. I can distinguish the particles by seeing their trajectories which are seprable

∴ Σ Probabilities

In case II: I am at S. I ~~can't~~ am seeing only a beam

∴ I can't distinguish. ∴ Σ Amplitudes

~~co~~ on comparing these equations

$$\langle +T|+S\rangle^* = \langle +S|+T\rangle \qquad \langle +S|0T\rangle = \langle 0T|+S\rangle^*$$

$$\therefore \quad \boxed{\langle a|x\rangle = \langle x|a\rangle^*} \Rightarrow \text{consequence of probability conservation}$$

EXPLAIN
~~STERN~~

STERN GERLAC filters in series

$$N \rightarrow \underset{S}{\begin{Bmatrix} + & \\ 0 & | \\ - & | \end{Bmatrix}} \xrightarrow{n} \underset{T}{\begin{Bmatrix} + & | \\ 0 & \\ - & | \end{Bmatrix}} \xrightarrow{\alpha_1 n} \underset{S}{\begin{Bmatrix} + & \\ 0 & | \\ - & | \end{Bmatrix}} \xrightarrow{\beta(\alpha_1 n)}$$

After comming out of T(0T) atoms don't remember that they were in (+S) state. The (+S) state information is over written by apparatus T.

Proof

$$N \rightarrow \underset{S}{\begin{Bmatrix} + & | \\ 0 & \\ - & | \end{Bmatrix}} \xrightarrow{n'} \underset{T}{\begin{Bmatrix} + & | \\ 0 & \\ - & | \end{Bmatrix}} \xrightarrow{\alpha_2 n'} \underset{S}{\begin{Bmatrix} + & \\ 0 & | \\ - & | \end{Bmatrix}} \xrightarrow{\beta(\alpha_2 n')}$$

Although the number differ, β, is same in both apparatus. the probability is same

If T filter passes only one beam the fraction that goes through second filter S depends only on the relative setup of T and completely independent of what precedes it.

BASE STATES

Any system can be represented in linear combination of states called BASE STATES.

Here we can define the system in these three base states. eg $(+S, 0S, -S)$ or $(+T, 0T, -T)$ etc.

The future of an atom in any single base state depends on nature of base state and independent of previous history (proved earlier)

$$\underset{S}{\begin{Bmatrix} + & \\ 0 & | \\ - & | \end{Bmatrix}} \qquad \underset{T}{\begin{Bmatrix} + & | \\ 0 & \\ - & | \end{Bmatrix}}$$

The atoms coming out of T $(0T)$ have no memory of being in (+S) state. If I wouldn't have filtered eg

$$\underset{S}{\begin{Bmatrix} + & \\ 0 & | \\ - & | \end{Bmatrix}} \qquad \underset{T}{\begin{Bmatrix} + & \\ 0 & \\ - & \end{Bmatrix}}$$

Now the $(+S)$ identity is maintained even after atoms go through T (I can prove this by putting S after T and showing no splitting).

REASON and FACT: Blocking / filtering destroys the information of particles

Some Important Experiments

$$\underset{S}{\begin{Bmatrix} + & \\ 0 & | \\ - & | \end{Bmatrix}} \xrightarrow{N} \underset{T}{\begin{Bmatrix} + & | \\ 0 & \\ - & | \end{Bmatrix}} \xrightarrow{\alpha N} \underset{S}{\begin{Bmatrix} + & \\ 0 & | \\ - & \end{Bmatrix}} \xrightarrow{\beta(\alpha N)}$$

$$\begin{Bmatrix} + & \\ 0 & | \\ - & | \end{Bmatrix} \xrightarrow{N} \begin{Bmatrix} + & | \\ 0 & \\ - & | \end{Bmatrix} \xrightarrow{\alpha N} \begin{Bmatrix} + & | \\ 0 & \\ - & | \end{Bmatrix} \xrightarrow{\gamma \alpha N}$$

$$\begin{Bmatrix} + \\ 0 \;| \\ - \;| \end{Bmatrix}_S \xrightarrow{N} \begin{Bmatrix} + \\ 0 \\ - \end{Bmatrix}_T \xrightarrow{N} \begin{Bmatrix} + \\ 0 \;| \\ - \;| \end{Bmatrix}_S \xrightarrow{N}$$

$$\begin{Bmatrix} + \\ 0 \;| \\ - \;| \end{Bmatrix}_S \xrightarrow{N} \begin{Bmatrix} + \\ 0 \\ - \end{Bmatrix}_T \xrightarrow{N} \begin{Bmatrix} + \;| \\ 0 \\ - \;| \end{Bmatrix}_S \xrightarrow{0}$$

$$\boxed{\begin{aligned} &\sum_i \langle +S \mid i \rangle \langle i \mid +S \rangle = 1 \\ &\sum_i \langle 0S \mid i \rangle \langle i \mid +S \rangle = 0 \end{aligned}} \quad \begin{aligned} &\equiv \langle +S \mid +S \rangle \\ &= \langle 0S \mid +S \rangle \end{aligned}$$

In this system I have used the base states of T to represent the amplitudes and stuff.

in general $\boxed{\langle \chi \mid \phi \rangle = \sum_{\substack{\text{base} \\ \text{states} \\ (i)}} \langle \chi \mid i \rangle \langle i \mid \phi \rangle}$

Physical Interpretation

$\langle \chi \mid \phi \rangle$ = amplitude of atoms in ϕ state ~~existing in~~ going through χ state

[small derivation

$|\phi\rangle = \sum a_i |i\rangle$ (amplitude) $\langle j \mid \phi \rangle = a_j$ $|\langle j \mid \phi \rangle|^2 = |a_j|^2$ (Probability)

amplitude of ϕ existing in base state $|j\rangle$]

$\langle i \mid \phi \rangle$ = amplitude of ϕ state to exist in i

$\langle \chi \mid i \rangle$ = amplitude of base state i to exist in χ

also we have seen $\boxed{\langle j \mid i \rangle = \delta_{ji}}$

$\therefore$ we get three rules

$$\boxed{\begin{aligned} &\text{I} \quad \langle j \mid i \rangle = \delta_{ji} \\ &\text{II} \quad \langle \chi \mid \phi \rangle = \sum_i \langle \chi \mid i \rangle \langle i \mid \phi \rangle \\ &\text{III} \quad \langle \phi \mid \chi \rangle = \langle \chi \mid \phi \rangle^* \end{aligned}}$$

The machinery of quantum mechanics

IDEA : The state of an atom is defined by 3 ~~numbers~~ amplitudes $\langle i | a \rangle$

The state of system is defined by 3 amplitudes $\langle x | i \rangle$

Thus $\langle x | a \rangle$ is going to tell the odds of atoms passing through the system

$$\underset{S}{\left\{ \begin{matrix} + & \\ 0 & | \\ - & | \end{matrix} \right\}} \qquad \left\{ A \right\} \qquad \underset{R}{\left\{ \begin{matrix} + & | \\ 0 & \\ - & | \end{matrix} \right\}}$$

complicated stuff going on

Amplitude of particle entering in +S state and emerging in (0R) state. or get through (0R)

$$a = \boxed{\langle 0R \,|\, A \,|\, +S \rangle} \qquad \langle \text{finish} \,|\, \text{through} \,|\, \text{start} \rangle$$

↓ General

$$\boxed{\langle x \,|\, A \,|\, a \rangle}$$

we choose some reframe base system (here T). Analogues to cartesian co ordinate system

$$\underset{S}{\left\{ \begin{matrix} + & \\ 0 & | \\ - & | \end{matrix} \right\}} \quad \underset{T}{\left\{ \begin{matrix} + \\ 0 \\ - \end{matrix} \right\}} \quad \left\{ A \right\} \quad \underset{T}{\left\{ \begin{matrix} + \\ 0 \\ - \end{matrix} \right\}} \quad \underset{R}{\left\{ \begin{matrix} + & | \\ 0 & \\ - & | \end{matrix} \right\}}$$

reframe

now ~~beca~~ I can define same amplitude a

$$\langle 0R | A | +S \rangle = a = \sum_{ij} \langle 0R | j \rangle \langle j | A | i \rangle \langle i | +S \rangle$$

$$\boxed{\langle x | A | a \rangle = \sum_{ij} \langle x | j \rangle \langle j | A | i \rangle \langle i | a \rangle}$$

The Apparatus is completely defined by nine amplitudes (7)
$\langle j | A | i \rangle$ which tell response of A w.r.t base
States of reframe base system (Here T)

	+	0	−
+	$\langle + \vert A \vert + \rangle$	$\langle + \vert A \vert 0 \rangle$	$\langle + \vert A \vert - \rangle$
0	$\langle 0 \vert A \vert 0 \rangle$	$\langle 0 \vert A \vert 0 \rangle$	$\langle 0 \vert A \vert - \rangle$
−	$\langle - \vert A \vert - \rangle$	$\langle - \vert A \vert 0 \rangle$	$\langle - \vert A \vert - \rangle$

once these 9 amplitudes are known and (x) (Q) are known in base States the result can be predicted

Now when C is composed of 2 or more system

$$\begin{Bmatrix} + \\ 0 \\ - \end{Bmatrix}_S \quad \{ C \} \quad \begin{Bmatrix} + \\ 0 \\ - \end{Bmatrix}_R$$

$$\{C\} = \{A\}\{B\}$$

I want to know 9 numbers

$$\langle j | C | i \rangle = ??$$

For this put reframe in between A and B

$$\{i\}\{A\}\begin{Bmatrix} + \\ 0 \\ - \end{Bmatrix}\{B\}\{j\} \equiv \{i\}\{C\}\{j\}$$

$$\sum_k \langle j | B | k \rangle \langle k | A | i \rangle = \langle j | C | i \rangle$$

This is what is known as matrix product

Transformation

I have Q state being represented in suppose (iS) state ($i = 0, +, -$) say. I want to express in (jT) state so

Transformation matrix

$$\boxed{\langle jT | Q \rangle = \sum_i \langle jT | iS \rangle \langle iS | Q \rangle}$$