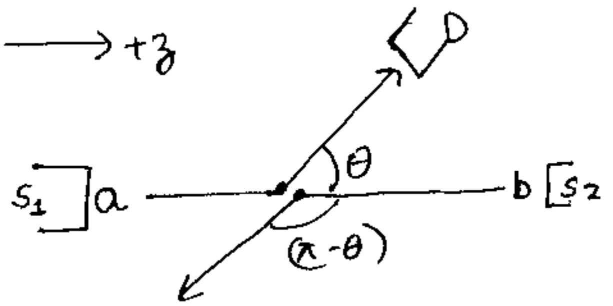
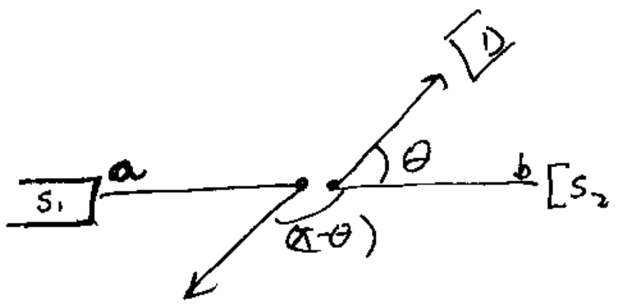
Collision of particles,

1) Distinguishable particles:



(I)



(II)

There is a detector $\underline{D}$ at an angle θ with respect to +z axis. The detector is able to distinguish weather particle a or b is striking.

The amplitude of particle a to collide at center [actually we are considering in frame of refrance of center of mass] and deflect at an angle θ such that it strikes D $= \langle \theta | a_{S1} \rangle = f(\theta)$ say

or the amplitude of case (I) to happen $= f(\theta)$
$= \langle \theta | a_{S1} \rangle$

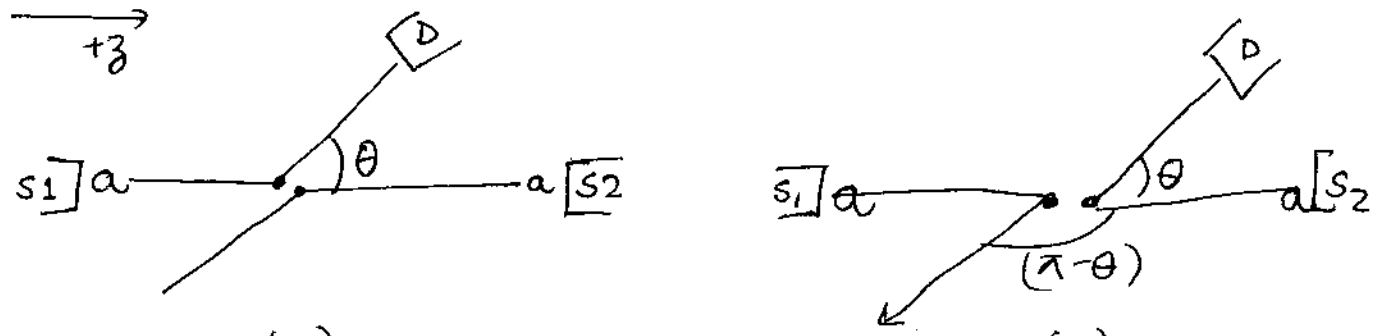
similarly the amplitude of 'a' to deflect at angle of $(\pi-\theta)$
= amplitude of 'b' to deflect at an angle of θ
= amplitude of case II to happen $= \langle \pi-\theta | a_{S1} \rangle$
$= \langle \theta | b_{S2} \rangle$
$= f(\pi-\theta) e^{i\delta}$

Thus by law of quantum mechanics the probability of detector at an angle 'θ' to detect a particle (any distinguishable a or b) $= |\langle \theta | a_{S1} \rangle|^2 + |\langle \pi-\theta | a_{S1} \rangle|^2$

$$P = |f(\theta)|^2 + |1|^2 |f(\pi-\theta)|^2$$

so far so good

2) Identical particles



(I) (II)

The Problem

Before collision I could mark the 'a' particles. The a particle comming from left [from S1] is ① and other one is ②. But I dont know which of the two cases is going to occur. Just by detecting particle 'a' at detector, I can in no way determine what had happened.

Solution

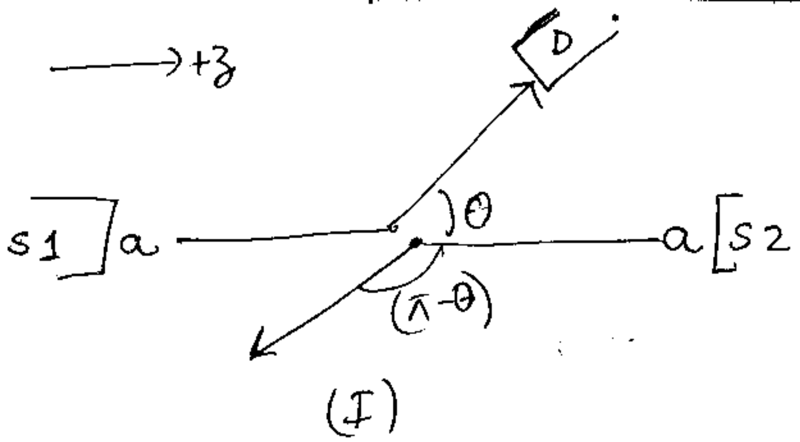
The amplitude of process (I) to occur $= \langle \theta \mid a_{\text{right}} \rangle = f(\theta)$ (a left)

The amplitude of process (II) to occur $= \langle \pi-\theta \mid a_{\text{right}} \rangle = e^{i\delta} f(\pi-\theta)$

[✱ I dont know why this factor of $e^{i\delta}$ comes. I strongly feel it is due to geometry. Or found the answer: It is exchange symmetry (indistinguishability) equate | rotality]

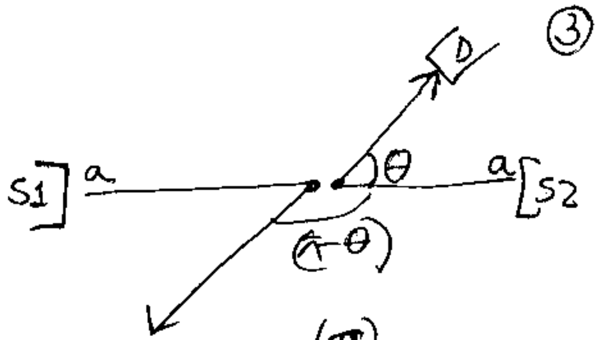
Thus the amplitude to detect particle a in detector D at angle θ	$= f(\theta) + e^{i\delta} f(\pi-\theta)$

So how can we evaluate $e^{i\delta}$? Does it have some allowed values or any value?



(I)

$f(\theta)$

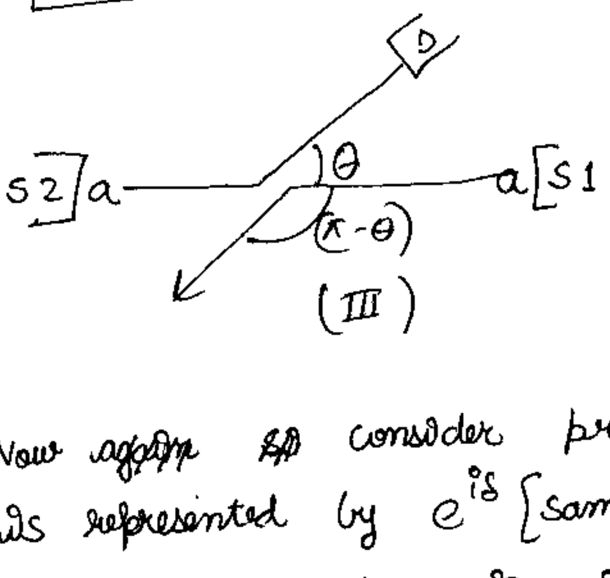


(II)

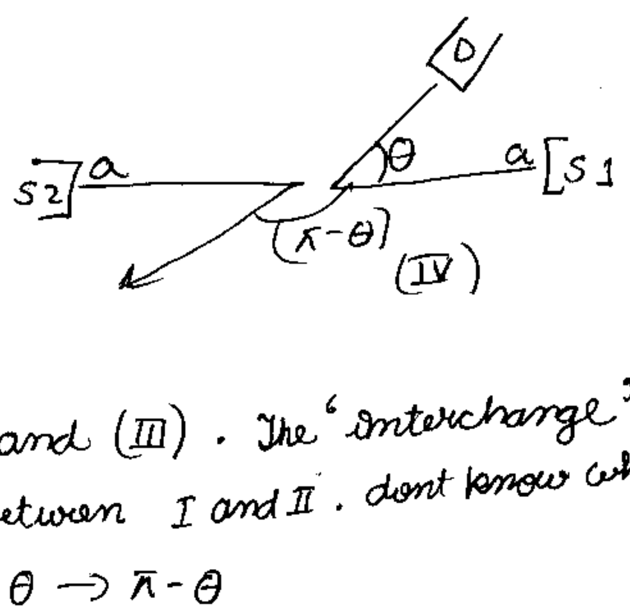
$e^{i\delta} f(\pi-\theta)$

ok lets consider if I interchange the identical 'a' particles. I <u>can</u> interchange because before collision I could mark them, differentiate them and thus change their role. It's that what kind of collision (I or II) is not going to happen is not in my hands (or is it ??)

INTERCHANGED ROLES [Sources are interchanged which introduce $e^{i\delta}$]



(III)



Now again consider process (I) and (III). The 'interchange' is represented by $e^{i\delta}$ [same phase between I and II. dont know why] and the amplitude is given by $\theta \to \pi - \theta$

The amplitude of III to happen $= e^{i\delta} \langle \theta | a_{\text{left}} \rangle$

" " IV " " $= e^{i\delta} \langle \pi-\theta | a \rangle$

$= e^{2i\delta} f(\pi-\theta)$

In interchanged case amplitude to get particle at D at θ $= e^{i\delta} f(\theta) + e^{2i\delta} f(\pi-\theta)$

If role is again interchanged we get amplitude as

$$= e^{2i\delta} f(\theta) + e^{3i\delta} f(\pi-\theta)$$

But we are back at first case thus amplitude must be exactly same

$$\therefore \quad e^{i\delta} f(\pi-\theta) + f(\theta) = e^{3i\delta} f(\pi-\theta) + e^{2i\delta} f(\theta)$$

$$e^{2i\delta} = 1$$

$$\boxed{e^{i\delta} = \pm 1}$$

+1 for Bosons (spin one)

-1 for fermions, also known as Pauli's exculsion principle (spin half)

for Bosons

$$\boxed{A = 2\left[f(\theta) + f(\pi-\theta)\right]}$$

also one can look on other way

A = (Amplitude direct) + (Amplitude Indirect)

* (2 can be left out because we can normalise it later)

for Fermions

$$\boxed{A = 2\left[f(\theta) - f(\pi-\theta)\right]}$$

A = (Amplitude direct) - (amplitude indirect)

<u>Approach</u>

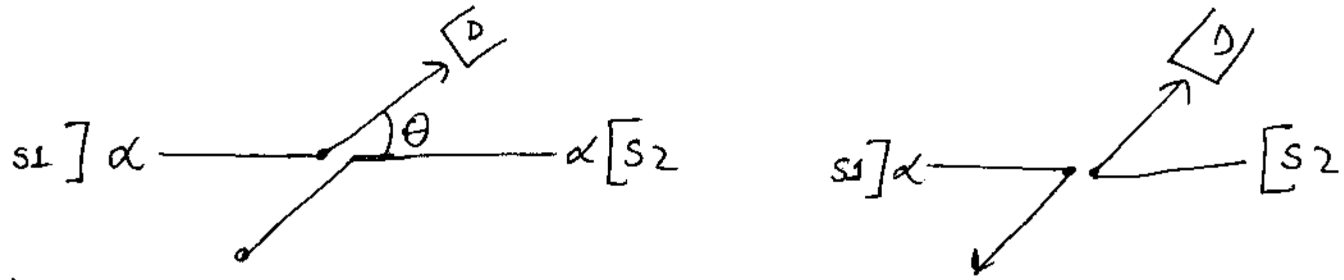
For identical particles consider both direct amplitude and indirect amplitude and add / subtract them according to their nature (Boson / fermion)

You can say that there is interferance of direct with "exchanged role" particles (+/- is again decided by particle)

[Note: we first make identical particals non identical by <u>marking</u> them and then destroy the marking by adding amplitude of interchanged roles]

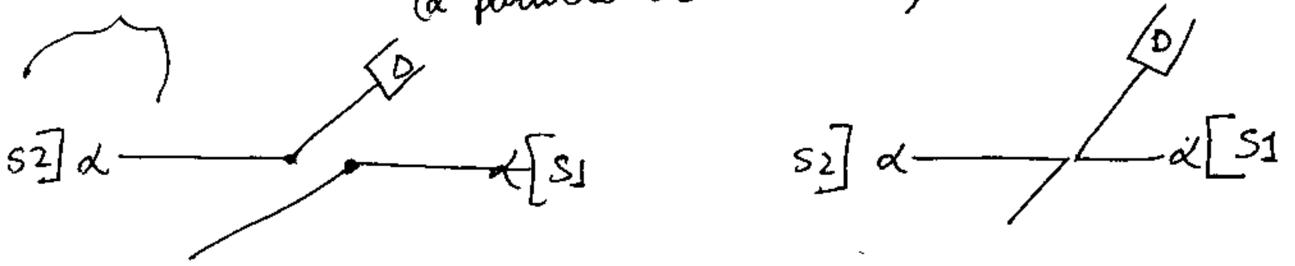
Case Study

For identical particles



Probability to detect α at D $= |f(\theta) + f(\pi-\theta)|^2$

(α particle is a boson)



For unpolarised electrons (spins are not changed during collisions is what we are going to assume)

see Feynman