Is Quantum Mechanical description of physical reality complete?

A report
for survey done by

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Chapter 1

EPR

In 1935 Einstein, Podolsky and Rosen (known as EPR) published an interesting paper. The title of the paper was

> Can the Quantum Mechanical description of the physical reality be considered complete?

This was the first time when strange nature of entanglement was brought into picture. By certain logical and mathematical arguments, EPR showed that the postulates of Quantum Mechanics with criteria of reality lead to the conclusion that the information given by a wavefunction is not complete.

First they defined the terms complete theory and reality of physical quantity of the theory.

- Complete Theory: The *necessary condition* for a theory to be complete is

  > every element of physical reality must have a counterpart in physical theory.

- Reality of Physical quantity: The *sufficient condition* for a physical quantity of a theory to have a reality is

  > if without any way disturbing the system, we can predict with certainty (i.e. with probability equal to unity), the value of the physical quantity then there exists an element of physical reality corresponding to this physical quantity.

They, then, illustrate the concept of reality in Quantum Mechanics. Consider a free particle with one degree of freedom. According to Quantum Mechanics, the particle is described by the state given by the wavefunction Ψ, which is the function of variables related to that particle. The wavefunction is assumed to contain all the information related to the particle. The theory further says that corresponding to each physical observable quantity A there is a self-adjoint operator $\hat{A}$ (denoted by same letter with hat). If Ψ is the eigenfunction of $\hat{A}$ then

$$\hat{A}\Psi = a\Psi,$$

where 'a' is the eigenvalue of $\hat{A}$. Whenever the particle is in state Ψ, then the value of physical quantity A is equal to a with probability equal to unity. Thus according to the criteria of reality, there is an element of reality corresponding to the physical quantity A.

For example consider $\Psi = \exp\left(\frac{i}{\hbar}p_0 x\right)$ where x is the coordinate (independent variable) related to the particle and p_0 is some constant. According to Quantum Mechanics, corresponding to physical observable quantity momentum (P), there is an operator

$$\hat{P} = -i\hbar\frac{\partial}{\partial x}.$$

Now it can be easily seen that

$$\hat{P}\Psi = p_0\Psi.$$

Thus according to EPR criteria of reality, there exists an element of reality corresponding to momentum (P), when particle is in state Ψ.

Also, in Quantum Mechanics, there exists an operator $\hat{Q}$ corresponding to a physical observable quantity coordinate (Q). Now for the same state Ψ

$$\hat{Q}\Psi = x\Psi \neq (\text{const})\Psi.$$

And now there exists a probability to find the particle between coordinates a and b defined by $P(a, b)$

$$P(a, b) = \int_a^b \Psi^*\Psi dx.$$

Here, according to Quantum Mechanics, one cannot predict the value of physical quantity coordinate with certainty and thus according to EPR criteria of reality, when the particle is in state Ψ, the coordinate has no reality. To obtain the definite value of coordinate, one has to perform some measurement which in turn changes the state of the particle. Thus, they conclude that

> when the momentum of the particle is known its coordinate has no reality.

More generally, it is shown by Quantum Mechanics, that if there are two operators $\hat{A}$ and $\hat{B}$ such that they dont commute i.e $\hat{A}\hat{B} \neq \hat{B}\hat{A}$, then the precise knowledge of former, precludes the knowledge of later. Furthermore, the attempt to determine the latter experimentally, will change the state of the system in a way which will destroy the knowledge of former.

According to EPR, there could be two alternatives

- the wavefunction is not giving complete information (The completeness condition demands every element of physical reality to have counterpart in theory. If both position and momentum operators (or any two non commuting operators) had corresponding element of physical reality, the wavefunction would have been such that we could extract the value of both with unit probability).

- if wavefunction is giving complete information then the physical quantities, corresponding to non commuting operators, don't have elements of reality *existing simultaneously*.

In the next part of the paper, EPR suggest a thought experiment. Consider a system of two systems (system 1 and system 2) having one degree of freedom each. From time $t = 0$ to $t = T$, they were allowed to interact and then, they are spatially separated with no interaction. The initial states of the individual systems were given/known. Thus one can calculate the state of the composite system (a system of the two systems) as a function of time from Schrodinger's equation. Let the state of the composite system after $t > T$ be denoted by Ψ and assume that it gives complete information of the composite system. If there is a physical observable A pertaining to system 1, then their is corresponding self-adjoint operator $\hat{A}$. Let $\upsilon_n(x_1)$ be the eigenfunction and u_n be the eigenvalue of $\hat{A}$. Then according to the spectral theorem, $\upsilon_n(x_1)$ form a basis set and any arbitrary function can be expanded in this basis. Wavefunction Ψ, considered as function of x_1, can be expanded in the basis set as

$$\Psi(x_1, x_2) = \sum_{n=1}^{\infty} \mathit{\Psi}_n(x_2)\upsilon_n(x_1),$$

where x_2 is the variable of system 2 and x_1 is variable of system 1. $\mathit{\Psi}_n(x_2)$ are the coefficients of the expansion (depending on variable of system 2)[1]. According to Quantum Mechanics, after the measurement of physical quantity A, system 1 will be left in a state whose wavefunction is given by $\upsilon_k(x_1)$ and system 2 will be left in the state with wavefunction $\mathit{\Psi}_k(x_2)$. This method is called *reduction of wave packet*. Thus the Ψ is reduced to $\mathit{\Psi}_k(x_2)\upsilon_k(x_1)$ after the measurement of observable A.

Similarly Ψ can be expanded in eigenfunctions of another operator $\hat{B}$ corresponding to physical observable quantity B as follows

$$\Psi(x_1, x_2) = \sum_{n=1}^{\infty} \mathit{\Phi}_n(x_2)\nu_n(x_1)$$

After the measurement of physical quantity B, the composite system is left in $\mathit{\Phi}_r(x_2)\nu_r(x_1)$. Now since the two systems (system 1 and system 2) were spatially separated during the measurement, system 2 can in no way recognize the type/kind of measurement done on system 1, or no real change can occur in system 2 owing to the type of measurement done on system 1. This is another way of saying that the two systems don't interact. Hence $\mathit{\Phi}_r(x_2)$ and $\Psi_k(x_2)$ should correspond to same reality. That is, these two wavefunctions can be assigned to the same reality of system 2. Now it may so happen that $\mathit{\Phi}_r(x_2)$ and $\Psi_k(x_2)$ be the eigenfunctions of non commuting operators (demonstrated in the following example). If so, then it is proved that the physical quantities corresponding to non commuting operators can have simultaneous reality.

Consider wavefunction of the composite system $\Psi(x_1, x_2)$[2]

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \exp\left(\frac{i}{\hbar}(x_1 - x_2 + x_0)p\right) dp, \tag{1.1}$$

[1] This is one of the examples of entangled systems.

[2] For continuous spectrum, summation is replaced by integrals like $\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \mathit{\Psi}_p(x_2)\upsilon_p(x_1)dp = \int_{-\infty}^{\infty} \mathit{\Phi}_x(x_2)\nu_x(x_1)dx$.

where x_0 is a constant. This is nothing but the well known Dirac-delta function $h\delta(x_1 - x_2 + x_0)$ expanded in momentum basis. It simple means that EPR is considering the situation when system 1 and 2 spatially separated such that $x_1 + x_0 = x_2$[3]. The above wavefunction can be expanded in the following ways

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \exp\left(\frac{-i}{\hbar}(x_2 - x_0)p\right) \exp\left(\frac{i}{\hbar}(x_1 p)\right) dp, \tag{1.2}$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} \exp\left(\frac{i}{\hbar}(x - x_2 + x_0)p\right) dp \right) \delta(x_1 - x) dx. \tag{1.3}$$

Let A be the momentum and B be the coordinate of system 1 also let P be the momentum and Q be the coordinate of system 2. Let $\hat{A}$[4], $\hat{B}$[5], $\hat{P}$[6] and $\hat{Q}$[7] be their corresponding operators. One can observe that for

$$\upsilon_p(x_1) = \exp\left(\frac{i}{\hbar}(x_1 p)\right),\ \nu_x(x_1) = \delta(x_1 - x) \text{ and}$$

$$\Psi_p(x_2) = \exp\left(\frac{-i}{\hbar}(x_2 - x_0)p\right) dp\ ,$$

$$\Phi_x(x_2) = \int_{-\infty}^{\infty} \exp\left(\frac{i}{\hbar}(x - x_2 + x_0)p\right) dp = h\delta(x - x_2 + x_0).$$

$\upsilon_p(x_1)$ is eigenfunction of $\hat{A}$ and $\nu_x(x_1)$ is eigenfunction of $\hat{B}$. Similarly $\Psi_p(x_2)$ is eigenfunction of $\hat{P}$ and $\Phi_x(x_2)$ is eigenfunction of $\hat{Q}$.

In this example one can clearly note that $\Psi_p(x_2)$ and $\Phi_x(x_2)$, which are supposed to correspond to same physical reality, are the eigenfunctions of non commuting operators $\hat{P}$ and $\hat{Q}$. The assumption that wavefunction provides complete information of the system has resulted in the real existence of position and momentum of a system simultaneously which is in violation to the earlier established result. Thus due to this contradiction

> one is forced to conclude that Quantum Mechanical description of physical reality is incomplete.

It can be argued that position and momentum of system 2 can't have *simultaneous* reality because the wavepacket reduction is not simultaneous. One can further argue that the position and momentum of system 2 are measured by performing non simultaneous measurements on system 1. Thus the $\hat{P}$ and $\hat{Q}$ don't have simultaneous reality.

But it should be noted that two systems are non interacting and spatially separated. The reality of physical entities pertaining to system 2 can't depend on *type* of measurement done on system 1. Thus both position and momentum of system 2 must be real simultaneously.

[3] Hence measurement done on one system can't possibly have any effect on other

[4] $\hat{A} = -i\hbar\partial/\partial x_1$

[5] $\hat{B} = \hat{x}$

[6] $\hat{P} = -i\hbar\partial/\partial x_2$

[7] $\hat{Q} = \hat{x}$

Bibliography

[1] A. Einstein, P. Podolsky and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Physical Review*, 47:777–780, 1935.